SAFE-FOLLOWING DISTANCES
BASED ON
THE CAR-FOLLOWING MODEL

Department of Highways
Thailand
Overview

- Introduction
- Methodology
- Data Collection
- Results
- Conclusions
Introduction

- More than 100,000 traffic accidents per year (12,000 deaths).
- Rear-end collisions ~ most common
- Aim of this research ~ reduce the severity and the number of road accidents.
- Derive safe following distance.
- Assist drivers with treatment marking.
Driving Task
Traffic Stability

- 1st decelerates.
- Others react.
- 7th and 8th collide.
Car-Following

- Notation:

- $x_n(t) - x_{n+1}(t) = \text{spacing (space headway)} = l_{n+1}(t) + \text{VL}$

- speed of vehicle $n$: $\frac{dx_n(t)}{dt} = \dot{x}_n(t)$

- acceleration (deceleration) of vehicle $n$: $\frac{d\dot{x}_n(t)}{dt} = \frac{d^2x_n(t)}{dt} = \ddot{x}_n(t)$

- $\ddot{x}_n(t) - \ddot{x}_{n+1}(t) = \dot{l}_{n+1}(t)$
\[ \ddot{x}_{n+1}(t + T) = \frac{\alpha[\dot{x}_{n+1}(t + T)]^m}{[x_n(t) - x_{n+1}(t)]^l} \{\dot{x}_n(t) - \dot{x}_{n+1}(t)\} \]

where \(\ddot{x}_{n+1}(t + T)\) is the acceleration of the \(n+1\)th car at time \(t + T\)
\(\dot{x}_{n+1}(t + T)\) is the velocity of the \(n+1\)th car at time \(t + T\)
\(x_n(t)\) is the distance of the \(n\)th car at time \(t\)
\(x_{n+1}(t)\) is the distance of the \(n+1\)th car at time \(t\)
\(\dot{x}_n(t)\) is the velocity of the \(n\)th car at time \(t\)
\(\dot{x}_{n+1}(t)\) is the velocity of the \(n+1\)th car at time \(t\)
\(\alpha\) is the sensitivity factor
\(m, l\) are constant
Methodology

- Calibrate the car-following model
- Analyze stability condition
- Derive safe following distance
Methodology

- Convert GM5th \( \Rightarrow \) Traffic Stream Model

\[ v = v_f \cdot \left[ 1 - \left( \frac{k}{k_j} \right)^{\gamma-1} \right]^{\frac{1}{1-\beta}} \]
Methodology

- Introduce error terms

\[ v_i = v_f \cdot \left[ 1 - \left( \frac{k_i}{k_f} \right)^{-1} \right]^{\frac{1}{1 - \beta}} + \varepsilon_i \]

\[ \varepsilon_i \sim N[0, \sigma^2] \]

\[ f(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\varepsilon_i^2} \]

- Log-likelihood

\[ \ln L(\beta, \gamma, v_f, k_f, \sigma^2) = \frac{-N}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{N} \left[ v_i - v_f \left[ 1 - \left( \frac{k_i}{k_f} \right)^{-1} \right]^{\frac{1}{1 - \beta}} \right]^2 \]
Stability Condition

• The Governing Equation of the car following model

\[ \ddot{x}_{n+1}(t+T) = \frac{\alpha [\dot{x}_{n+1}(t+T)]^m}{[x_n(t) - x_{n+1}(t)]^l} \{\dot{x}_n(t) - \dot{x}_{n+1}(t)\} \]

• Perturb the equilibrium solution with a small deviation term

\[ X_n(t) = b \cdot n + v \cdot t + \varepsilon(n,t) \]

where \[ \varepsilon(n,t) = f_n \cdot e^{i\alpha t} \]
Stability Condition

\[ \frac{\alpha \cdot [v]^\beta}{b^\gamma} \cdot T \leq \frac{1}{2} \]

Where

- \( v \) = prevailing speed
- \( T \) = reaction time
- \( b \) = distance headway
Case Study

- National Highway 7 (Chon Buri Motorway)
- Located at km 50+000 toward Chon Buri
- Traffic volumes ~heavy during weekend morning and afternoon peak periods (ADT ~ 60,000)
Study Site
Detector Placement
### Calibration Result

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$</td>
<td>33.646</td>
<td>2.238</td>
<td>15.034</td>
</tr>
<tr>
<td>$v_f$</td>
<td>95.716</td>
<td>0.080</td>
<td>1,196.450</td>
</tr>
<tr>
<td>$k_j$</td>
<td>116.067</td>
<td>1.482</td>
<td>78.318</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4.510</td>
<td>0.042</td>
<td>107.143</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>0.00002</td>
<td>49500.000</td>
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</tbody>
</table>
### Comparison of Recommended Following Distances

<table>
<thead>
<tr>
<th>Speed (km/h)</th>
<th>Safe Following Distance (m)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pipes**</td>
<td>Forbes**</td>
</tr>
<tr>
<td>80</td>
<td>30.00</td>
<td>38.33</td>
</tr>
<tr>
<td>90</td>
<td>33.13</td>
<td>42.50</td>
</tr>
<tr>
<td>100</td>
<td>36.25</td>
<td>46.67</td>
</tr>
<tr>
<td>110</td>
<td>39.38</td>
<td>50.83</td>
</tr>
<tr>
<td>120</td>
<td>42.50</td>
<td>55.00</td>
</tr>
<tr>
<td>130</td>
<td>45.63</td>
<td>59.17</td>
</tr>
</tbody>
</table>

*Assume average vehicle length of 5 meters.

Recommended following distances
Following Distances Based on Different Car-Following Models

- Pipes'
- Forbes'
- Calibrated GM
- 2-second Rule
Speed-Flow Curve of the Traffic on the National Highway 7 toward Chon Buri
“DOT” Tailgating Treatment
Sign and Pattern Layout
**Typical “DOT” Marking**

![Diagram of DOT marking with dimensions A and B, where Width/Length = 1:3 ratio.]

### Typical Marking

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A (m)</th>
<th>B (m)</th>
<th>Area (sq m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorway</td>
<td>2.25</td>
<td>0.75</td>
<td>1.33</td>
</tr>
</tbody>
</table>

*Based on standard oblong pavement markings referenced in the MUTCD*
When vehicles approach the "DOT" and the driver perceives his vehicle as touching the first marking the front of the vehicle is actually 4.5 meters away from it as shown here.

<table>
<thead>
<tr>
<th>Comprehension Time</th>
<th>5 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRT Time</td>
<td>2.5 sec</td>
</tr>
<tr>
<td>Adjustment Time</td>
<td>20 sec</td>
</tr>
<tr>
<td>Effective Time</td>
<td>60 sec</td>
</tr>
<tr>
<td>Vehicle Correlation</td>
<td>4.6 meters</td>
</tr>
<tr>
<td>Vehicle Length</td>
<td>5.0 meters</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Posted Speed (km/h)</th>
<th>Recommended Following Distance (meters)</th>
<th>S Marking Spacing (meters)</th>
<th>Minimum Marking in Pattern</th>
<th>L Min Pattern Length (meters)</th>
<th>X Pattern Spacing (meters)</th>
<th>Capacity (pc/in/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>50</td>
<td>41</td>
<td>15</td>
<td>574</td>
<td>1300</td>
<td>1440</td>
</tr>
<tr>
<td>90</td>
<td>52</td>
<td>43</td>
<td>16</td>
<td>645</td>
<td>1500</td>
<td>1560</td>
</tr>
<tr>
<td>100</td>
<td>53</td>
<td>44</td>
<td>18</td>
<td>748</td>
<td>1600</td>
<td>1700</td>
</tr>
<tr>
<td>110</td>
<td>54</td>
<td>45</td>
<td>19</td>
<td>810</td>
<td>1800</td>
<td>1830</td>
</tr>
<tr>
<td>120</td>
<td>55</td>
<td>46</td>
<td>20</td>
<td>874</td>
<td>2000</td>
<td>1960</td>
</tr>
</tbody>
</table>

*Remark*
1. If the observed volume exceeds the capacity provided in this table, the "DOT" tailgating treatment should not be installed.
2. The "DOT" tailgating treatment should only be installed to the location where rear-end collisions due to aggressive driving behaviors are frequent.
CONCLUSIONS

• MLE => calibrate => GM 5th car-following model
• Assist drivers => Following distance for speed range 80 - 120 km/h
FUTURE RESEARCH

- More Data Collection
- Reaction Time $\Rightarrow$ determination
- More Field Applications $\Rightarrow$ validation
THANK YOU